Kinds Are Calling Conventions

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Efficient Function Calls

Parameter Passing Techniques

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- Representation — What & Where?

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Goal: An IL with ***un**restricted* η for functions, along with *restricted* β for other types

Static Arity

In an Intermediate Language

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  - $\lambda x . f\ x =_\eta f : a \rightsquigarrow b$ *unconditionally*
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  - $(\lambda x . x + x)\ (\mathit{fact}\ 10^6)$ does not recompute $\mathit{fact}\ 10^6$

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- Application may still be *restricted* for efficiency, like source functions
  - $(\lambda x . x + x)\ (\mathit{fact}\ 10^6)$ does not recompute $\mathit{fact}\ 10^6$
- With full η, types express arity — just count the arrows
  - $f : \mathit{Int} \rightsquigarrow \mathit{Bool} \rightsquigarrow \mathit{String}$ has arity 2, no matter f’s definition

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When Partial Application Matters

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  - map (App (f3' 100)) [1..10^6] computes 'expensive 100' only once ☺

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Clos :: (Int ~> Int) ~> {Int ~> Int}   App  :: {Int ~> Int} ~> Int ~> Int
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- `x = x’` by η, and `x’` always follows call-by-name order!
- Primitive functions are never just *evaluated*; they are always *called*

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And Static Compilation

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- But what if a = Bool ~> Bool?
  - `f :: Int ~> Int ~> Bool ~> Bool` has arity 3...
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- How to statically compile? Is ‘`g 5`’ a call? A partial application?

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- `f :: a ~> b :: TYPE Ptr Call[2]` has arity 2

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- `f :: a ~> b :: TYPE Ptr Call[2]` has arity 2
- `x :: a :: TYPE Ptr c` is represented as a pointer

Even More

In the Paper

- Levity Polymorphism
  - For when evaluation strategy doesn't matter
- Compiling Source → Intermediate → Target
  - Via kind-directed η-expansion and register assignment
- Type system for ensuring static compilation
  - Of definitions with arity, levity, and representation polymorphism

Kinds capture the details of efficient calling conventions